

INTRODUCTION

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1. Overview

0001 Besides the book by Laumon and Moret-Bailly, see [LMB00], and the work (in progress) by Fulton et al, we think there is a place for an open source textbook on algebraic stacks and the algebraic geometry that is needed to define them. The Stacks Project attempts to do this by building the foundations starting with commutative algebra and proceeding via the theory of schemes and algebraic spaces to a comprehensive foundation for the theory of algebraic stacks.

We expect this material to be read online as a key feature are the hyperlinks giving quick access to internal references spread over many different pages. If you use an embedded pdf or dvi viewer in your browser, the cross file links should work.

This project is a collaborative effort and we encourage you to help out. Please email any typos or errors you find while reading or any suggestions, additional material, or examples you have to stacks.project@gmail.com. You can download a tarball containing all source files, extract, run make, and use a dvi or pdf viewer locally. Please feel free to edit the LaTeX files and email your improvements.

2. Attribution

06LB The scope of this work is such that it is a daunting task to attribute correctly and succinctly all of those mathematicians whose work has led to the development of the theory we try to explain here. We hope eventually to generate enough community interest to find contributors willing to write sections with historical remarks for each and every chapter.

This is a chapter of the Stacks Project, version 1be5a612, compiled on Jul 19, 2017.

Those who contributed to this work are listed on the title page of the book version of this work and online. Here we would like to name a selection of major contributions:

- (1) Jarod Alper wrote Guide to Literature.
- (2) Bhargav Bhatt wrote the initial version of *Étale Morphisms of Schemes*.
- (3) Bhargav Bhatt wrote the initial version of *More on Algebra*, Section 78.
- (4) Kiran Kedlaya contributed the initial writeup of *Descent*, Section 4.
- (5) The initial versions of
 - (a) *Algebra*, Section 27,
 - (b) *Injectives*, Section 2, and
 - (c) the chapter *Fields*
 are from The CRing Project, courtesy of Akhil Mathew et al.
- (6) Alex Perry wrote the material on projective modules, Mittag-Leffler modules, including the proof of *Algebra*, Theorem 94.5.
- (7) Alex Perry wrote *Formal Deformation Theory*.
- (8) Thibaut Pugin, Zachary Maddock and Min Lee took course notes which formed the basis for *Étale Cohomology*.
- (9) David Rydh has contributed many helpful comments, pointed out several mistakes, helped out in an essential way with the material on residual gerbes, and was the originator for the material in *More on Groupoids in Spaces*, Sections 12 and 15.
- (10) Burt Totaro contributed *Examples*, Sections 56, 57, and *Properties of Stacks*, Section 12.
- (11) The material in the chapter *Pro-étale Cohomology* is taken from a paper by Bhargav Bhatt and Peter Scholze.
- (12) Bhargav Bhatt contributed *Examples*, Sections 63 and 67.
- (13) Ofer Gabber found mistakes, contributed corrections and he contributed *Varieties*, Lemma 7.17, *Formal Spaces*, Lemma 9.5, the material in *More on Groupoids*, Section 15, the main result of *Properties of Spaces*, Section 16, and the proof of *More on Flatness*, Proposition 25.13.
- (14) János Kollár contributed *Algebra*, Lemma 118.2 and *Local Cohomology*, Proposition 4.7.
- (15) Kiran Kedlaya wrote the initial version of *More on Algebra*, Section 79.
- (16) Matthew Emerton, Toby Gee, and Brandon Levin contributed some results on thickenings, in particular *More on Morphisms of Stacks*, Lemmas 3.7, 3.8, and 3.9.
- (17) Lena Min Ji wrote the initial version of *More on Algebra*, Section 99.
- (18) Matthew Emerton and Toby Gee wrote the initial versions of *Geometry of Stacks*, Sections 3 and 5.

3. Other chapters

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| <p>Preliminaries</p> <ol style="list-style-type: none"> (1) Introduction (2) Conventions (3) Set Theory (4) Categories (5) Topology (6) Sheaves on Spaces | <ol style="list-style-type: none"> (7) Sites and Sheaves (8) Stacks (9) Fields (10) Commutative Algebra (11) Brauer Groups (12) Homological Algebra (13) Derived Categories |
|---|--|

- (14) Simplicial Methods
 - (15) More on Algebra
 - (16) Smoothing Ring Maps
 - (17) Sheaves of Modules
 - (18) Modules on Sites
 - (19) Injectives
 - (20) Cohomology of Sheaves
 - (21) Cohomology on Sites
 - (22) Differential Graded Algebra
 - (23) Divided Power Algebra
 - (24) Hypercoverings
- Schemes
- (25) Schemes
 - (26) Constructions of Schemes
 - (27) Properties of Schemes
 - (28) Morphisms of Schemes
 - (29) Cohomology of Schemes
 - (30) Divisors
 - (31) Limits of Schemes
 - (32) Varieties
 - (33) Topologies on Schemes
 - (34) Descent
 - (35) Derived Categories of Schemes
 - (36) More on Morphisms
 - (37) More on Flatness
 - (38) Groupoid Schemes
 - (39) More on Groupoid Schemes
 - (40) Étale Morphisms of Schemes
- Topics in Scheme Theory
- (41) Chow Homology
 - (42) Intersection Theory
 - (43) Picard Schemes of Curves
 - (44) Adequate Modules
 - (45) Dualizing Complexes
 - (46) Duality for Schemes
 - (47) Discriminants and Differents
 - (48) Local Cohomology
 - (49) Algebraic Curves
 - (50) Resolution of Surfaces
 - (51) Semistable Reduction
 - (52) Fundamental Groups of Schemes
 - (53) Étale Cohomology
 - (54) Crystalline Cohomology
 - (55) Pro-étale Cohomology
- Algebraic Spaces
- (56) Algebraic Spaces
 - (57) Properties of Algebraic Spaces
- (58) Morphisms of Algebraic Spaces
 - (59) Decent Algebraic Spaces
 - (60) Cohomology of Algebraic Spaces
 - (61) Limits of Algebraic Spaces
 - (62) Divisors on Algebraic Spaces
 - (63) Algebraic Spaces over Fields
 - (64) Topologies on Algebraic Spaces
 - (65) Descent and Algebraic Spaces
 - (66) Derived Categories of Spaces
 - (67) More on Morphisms of Spaces
 - (68) Flatness on Algebraic Spaces
 - (69) Groupoids in Algebraic Spaces
 - (70) More on Groupoids in Spaces
 - (71) Bootstrap
 - (72) Pushouts of Algebraic Spaces
- Topics in Geometry
- (73) Quotients of Groupoids
 - (74) More on Cohomology of Spaces
 - (75) Simplicial Spaces
 - (76) Duality for Spaces
 - (77) Formal Algebraic Spaces
 - (78) Restricted Power Series
 - (79) Resolution of Surfaces Revisited
- Deformation Theory
- (80) Formal Deformation Theory
 - (81) Deformation Theory
 - (82) The Cotangent Complex
 - (83) Deformation Problems
- Algebraic Stacks
- (84) Algebraic Stacks
 - (85) Examples of Stacks
 - (86) Sheaves on Algebraic Stacks
 - (87) Criteria for Representability
 - (88) Artin's Axioms
 - (89) Quot and Hilbert Spaces
 - (90) Properties of Algebraic Stacks
 - (91) Morphisms of Algebraic Stacks
 - (92) Limits of Algebraic Stacks
 - (93) Cohomology of Algebraic Stacks
 - (94) Derived Categories of Stacks
 - (95) Introducing Algebraic Stacks
 - (96) More on Morphisms of Stacks
 - (97) The Geometry of Stacks
- Topics in Moduli Theory
- (98) Moduli Stacks
 - (99) Moduli of Curves
- Miscellany

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|---------------------------|--------------------------------------|
| (100) Examples | (105) Obsolete |
| (101) Exercises | (106) GNU Free Documentation License |
| (102) Guide to Literature | (107) Auto Generated Index |
| (103) Desirables | |
| (104) Coding Style | |

References

- [LMB00] Gérard Laumon and Laurent Moret-Bailly, *Champs algébriques*, Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge., vol. 39, Springer-Verlag, 2000.